

- a) Let $G := G[X, Y]$ be a bipartite graph, M a matching in G , and U the set of vertices in X not covered by M . Show that:
- i) for any subset S of X , $|U| \geq |S| - |N(S)|$,
 - ii) $|U| = |S| - |N(S)|$ for some subset S of X if and only if M is a maximum matching of G .
- b) Deduce the following generalization of Hall's Theorem (16.4):

The matching number of a bipartite graph $G := G[X, Y]$ is given by:

$$\alpha' = |X| - \max\{|S| - |N(S)| : S \subseteq X\}$$

This expression for α' is known as the *König-Ore Formula*