

For a connected graph G , define $\sigma(G) := \sum\{d(u, v) : u, v \in V\}$.

1. Let G be a connected graph. For $v \in V$, let T_v be a BFS-tree of G rooted at v . Show that $\sum_{v \in V} \sigma(T_v) \leq 2(n-1)\sigma(G)$.
2. Deduce that every connected graph G has a spanning tree T such that $\sigma(T) \leq 2(1 - \frac{1}{n})\sigma(G)$. (R.C. ENTRINGER, D.J. KLEITMAN AND L. SZÉKELY)