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Flows in Networks

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7.1 Transportation Networks

Transportation networks that are used to ship commodities from their production centres to their markets can be most effectively analysed when viewed as digraphs that possess additional structure. The resulting theory has a wide range of interesting applications and ramifications. We present here the basic elements of this important topic.

A *network* $N := N(x, y)$ is a digraph D (the *underlying digraph* of N) with two distinguished vertices, a *source* x and a *sink* y, together with a nonnegative real-valued function c defined on its arc set A. The vertex x corresponds to a production centre, and the vertex y to a market. The remaining vertices are called *intermediate vertices*, and the set of these vertices is denoted by I. The function c is the *capacity function* of N and its value on an arc a the *capacity* of a. The capacity of an arc may be thought of as representing the maximum rate at which a commodity can be transported along it. It is convenient to allow arcs of infinite capacity, along which commodities can be transported at any desired rate. Of course, in practice, one is likely to encounter transportation networks with several