5

Nonseparable Graphs

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5.1 Cut Vertices

In Chapter 3, we introduced the notion of a cut edge and discussed various properties of connected graphs without cut edges. Here, we consider the analogous notion for vertices. There are, in fact, two closely related notions, that of a cut vertex and that of a separating vertex.

A *cut vertex* of a graph G is a vertex v such that $c(G-v) > c(G)$. In particular, a cut vertex of a connected graph is a vertex whose deletion results in a disconnected graph. This notion is illustrated in Figure 5.1, the cut vertices being indicated by solid dots.

By Exercise 3.1.3, a graph is connected if any two of its vertices are connected by a path. Connected graphs without cut vertices have a stronger property, described in the theorem below. Two distinct paths are *internally disjoint* if they have no internal vertices in common.

Theorem 5.1 *A connected graph on three or more vertices has no cut vertices if and only if any two distinct vertices are connected by two internally disjoint paths.*