
Nonseparable Graphs

Contents

5.1	Cut Vertices	117
5.2	Separations and Blocks	119
	NONSEPARABLE GRAPHS	119
	BLOCKS	120
	PROOF TECHNIQUE: SPLITTING OFF EDGES	122
5.3	Ear Decompositions	125
	STRONG ORIENTATIONS	126
5.4	Directed Ear Decompositions	129
5.5	Related Reading	133
	EVEN CYCLE DECOMPOSITIONS	133
	MATROIDS AND NONSEPARABILITY	133

5.1 Cut Vertices

In Chapter 3, we introduced the notion of a cut edge and discussed various properties of connected graphs without cut edges. Here, we consider the analogous notion for vertices. There are, in fact, two closely related notions, that of a cut vertex and that of a separating vertex.

A *cut vertex* of a graph G is a vertex v such that $c(G-v) > c(G)$. In particular, a cut vertex of a connected graph is a vertex whose deletion results in a disconnected graph. This notion is illustrated in Figure 5.1, the cut vertices being indicated by solid dots.

By Exercise 3.1.3, a graph is connected if any two of its vertices are connected by a path. Connected graphs without cut vertices have a stronger property, described in the theorem below. Two distinct paths are *internally disjoint* if they have no internal vertices in common.

Theorem 5.1 *A connected graph on three or more vertices has no cut vertices if and only if any two distinct vertices are connected by two internally disjoint paths.*