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## Electrical Networks

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### 20.1 Circulations and Tensions

We saw in Section 2.6 that the even subgraphs and edge cuts of a graph form vector spaces over  $GF(2)$ , namely the cycle and bond spaces of the graph. Here, we consider analogous vector spaces over the reals, and more generally over any field. Throughout this section,  $D$  denotes a connected (though not necessarily strongly connected) digraph and  $T$  a spanning tree of  $D$ .

#### THE CIRCULATION AND TENSION SPACES

In Section 7.3, we defined a *circulation* in a digraph  $D$  as a function  $f : A \rightarrow \mathbb{R}$  which satisfies the conservation condition at every vertex: