
Hamilton Cycles

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18.1 Hamiltonian and Nonhamiltonian Graphs

Recall that a path or cycle which contains every vertex of a graph is called a *Hamilton path* or *Hamilton cycle* of the graph. Such paths and cycles are named after Sir William Rowan Hamilton, who described, in a letter to his friend Graves in 1856, a mathematical game on the dodecahedron (Figure 18.1a) in which one