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## The Probabilistic Method

### Contents

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<b>13.1 Random Graphs</b> .....	<b>329</b>
INDEPENDENT EVENTS .....	331
RANDOM VARIABLES .....	331
<b>13.2 Expectation</b> .....	<b>333</b>
LINEARITY OF EXPECTATION .....	333
THE CROSSING LEMMA .....	334
ASYMPTOTIC NOTATION .....	335
MARKOV'S INEQUALITY .....	336
<b>13.3 Variance</b> .....	<b>342</b>
CHEBYSHEV'S INEQUALITY .....	342
STABILITY NUMBERS OF RANDOM GRAPHS .....	343
<b>13.4 Evolution of Random Graphs</b> .....	<b>347</b>
THRESHOLD FUNCTIONS .....	347
BALANCED GRAPHS .....	347
THE GIANT COMPONENT .....	349
<b>13.5 The Local Lemma</b> .....	<b>350</b>
TWO-COLOURABLE HYPERGRAPHS .....	353
EVEN CYCLES IN DIRECTED GRAPHS .....	353
LINEAR ARBORICITY .....	354
<b>13.6 Related Reading</b> .....	<b>355</b>
PROBABILISTIC MODELS .....	355
SHARP THRESHOLD FUNCTIONS .....	356
CONCENTRATION INEQUALITIES .....	356

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### 13.1 Random Graphs

As mentioned in Section 12.3, the lower bound on Ramsey numbers given in Theorem 12.8 was obtained by means of a proof technique known as the *probabilistic method*. Roughly speaking, this technique is based on an understanding of how