4

Trees

Contents

4.1 Forests and Trees

Recall that an *acyclic* graph is one that contains no cycles. A connected acyclic graph is called a *tree*. The trees on six vertices are shown in Figure 4.1. According to these definitions, each component of an acyclic graph is a tree. For this reason, acyclic graphs are usually called *forests*.

In order for a graph to be connected, there must be at least one path between any two of its vertices. The following proposition, an immediate consequence of Exercise 2.2.12, says that trees are the connected graphs which just meet this requirement.

Proposition 4.1 *In a tree, any two vertices are connected by exactly one path.* □

Following Diestel (2005), we denote the unique path connecting vertices x and y in a tree T by xTy.

By Theorem 2.1, any graph in which all degrees are at least two contains a cycle. Thus, every tree contains a vertex of degree at most one; moreover, if the tree is nontrivial, it must contain a vertex of degree exactly one. Such a vertex is called a *leaf* of the tree. In fact, the following stronger assertion is true (Exercise 2.1.2).